$\mathbf{SU}_q(n)$ **Gauge Theory**

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Abstract

A field theory with local transformations belonging to the quantum group $\mathrm{SU}_q(n)$ is defined on a classical spacetime, with gauge potentials belonging to a quantum Lie algebra. Gauge transformations are defined for the potentials which lead to the appropriate quantum-group transformations for field strengths and covariant derivatives, defined for all elements of $\mathrm{SU}_q(n)$ by means of the adjoint action. This guarantees a non-trivial deformation. Gauge-invariant commutation relations are identified.

1   Introduction

A number of authors [1–11, 13, 14] have proposed q-deformations of gauge field theory. One motivation for this is the prospect of a theory with as large a set of invariances as conventional gauge theory—and therefore, hopefully, the same renormalisability properties—but in which the symmetry of physical predictions is broken without the intervention of extra fields such as the Higgs field. Another motivation is the hope that a consistent quantum theory of gravity may take such a form. The most successful of these theories [4, 7, 8] have been those in which the

structure of space-time is made non-commutative. This is an interesting feature if one's aim is to construct a theory of quantum gravity; but for the first motivation mentioned above, it would be more natural to construct the theory on a classical space-time. All attempts so far to do this have failed. In some cases [7, 14] the theory does not have a good classical limit as $q \to 1$. This is not surprising for a theory based on a bicovariant calculus on a quantum group, as such calculi do not normally have a good classical limit; only for the quantum groups $\mathrm{GL}_q(n)$ does one obtain a Lie algebra with the same dimension as the classical one, and therefore a deformed gauge theory with the same number of gauge fields as the classical theory. Other theories [9, 10, 11] have the opposite problem: the classical limit exists, but the deformed theory never gets away from it, remaining always isomorphic to it. The reason for this, as was explained by Brzezinski and Majid [3], is that such theories use a notion of gauge transformation modelled too closely on the classical form of conjugation, namely $F \mapsto gFg^{-1}$ where g is an invertible element of the quantum group, instead of the adjoint action $F \mapsto g_{(1)}FS(g_{(2)})$ which is more appropriate for a quantum group with its Hopf algebra structure (and, indeed, for a classical group when one wants to consider non-invertible gauge transformations such as infinitesimal ones). This adjoint type of gauge transformation has been thought to be impossible because "there is no way to define F from [the gauge field] to transform as desired". In this paper a way is found.

The basis of the theory presented here is a new notion of quantum Lie algebra [12] which does not require the existence of a bicovariant differential calculus. This leads to a very natural definition of a gauge transformation for gauge potentials in a classical space-time such that the transformation at each point is an element of a quantum group instead of a group, and hence to a q-deformed gauge theory based entirely on the Hopf algebra structure of a quantum groups. The theory contains gauge potentials whose gauge transformations imply the appropriate Hopf adjoint-type transformation properties for the field strengths. The required quantum Lie algebra is known to exist for the quantum groups $\mathrm{SU}_q(n)$, and is conjectured to exist for the q-deformations of all simple Lie groups.

The geometrical picture to bear in mind in considering this theory is similar to the geometry of fibre bundles underlying classical gauge theory; it is a picture of vector bundles over a classical manifold, the structure group by which the vectors transform being replaced by a Hopf algebra (generalising the group algebra of the structure group or—what is conceptually almost the same thing—the enveloping algebra of the Lie algebra of the structure group). If this Hopf algebra is the quantised enveloping algebra $\mathrm{U}_q(\mathbf{su}(n))$ it can also be regarded as an enveloping algebra, being generated by a quantum Lie algebra [12]; this makes it possible to define a connection in the Hopf vector bundle with a similar geometrical meaning to the connection in a vector bundle with a classical structure group. This geometry is classical in the sense that it has points in the base manifold; it is not necessary, as it usually is in non-commutative geometry, to renounce points in

favour of functions.

The resulting gauge theory can only be considered as a quantum field theory; it cannot be defined as a classical field theory because the components of the fields cannot be taken to commute. For consistency with the gauge transformations, they must satisfy q-commutation relations. In the case of the gauge potentials, it is not even possible to impose the full set of q-commutation relations; roughly speaking, half of the commutators must be unspecified. This feature was also found by Castellani [6] in his $U_q(n)$ gauge theory. Unlike Castellani, we do not need to suppose that the parameters of our gauge transformations are non-commuting objects.

The theory has the desired feature of a set of gauge transformations as large as in classical gauge theory, while exhibiting symmetry breaking in its physical predictions. However, this symmetry breaking is not sufficient to provide a non-zero mass for the gauge fields without some further mechanism such as a Higgs field.

2 q-Gauge Transformations

Let $\mathcal{H} = U_q(\mathbf{g})$ be the quantised enveloping algebra of the N-dimensional Lie algebra $\mathbf{g}$. This is a Hopf algebra, with a coproduct $\Delta x = \sum x_{(1)} \otimes x_{(2)}$ and an antipode $S(x)$, and it acts on itself by the adjoint action

$$\mathrm{ad}x(y) = x_{(1)} y S(x_{(2)}) \tag{1}$$

where, as is customary, we have omitted the $\sum$ sign in the sum over the terms in the coproduct on the right-hand side. A *quantum Lie algebra* $\mathbf{g}_q$ is an N-dimensional subspace of $\mathcal{H}$ with the following properties:

1. $\mathbf{g}_q$ is invariant under the adjoint action, so that the quantum Lie bracket

$$[x, y]_q = \mathrm{ad}x(y) \in \mathbf{g}_q \tag{2}$$

   is defined for all $x, y \in \mathbf{g}_q$.

2. $\mathbf{g}_q$ generates $\mathcal{H}$ as an algebra.

3. The elements of $\mathbf{g}_q$ satisfy relations of the form

$$X_i X_j - \sigma_{ij}^{kl} X_k X_l = C[X_i, X_j] \tag{3}$$

   where C is a central element of $\mathcal{H}$ (a function of the Casimirs of $\mathbf{g}_q$), the X_i are basis elements of $\mathbf{g}_q$, and σ_{ij}^{kl} is an $N^2 \times N^2$ matrix which has 1 as an eigenvalue (a deformation of the classical flip operator: $\sigma_{ij}^{kl} = \delta_i^l \delta_j^k$ when $q = 1$).

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4. The quantum Lie bracket (2) is antisymmetric with respect to σ in the sense that

$$t^{ij}[X_i, X_j]_q = 0 \qquad \text{whenever} \qquad t^{ij}\sigma_{ij}^{kl} = t^{kl}. \qquad (4)$$

It is known [12] that a quantum Lie algebra $\mathbf{g}_q$ exists for $\mathbf{g} = \mathbf{su}(n)$ (though the sense in which it generates $U_q(\mathbf{g})$ has yet to be rigorously defined) and that in this case the coproducts of the elements of $\mathbf{g}_q$ are of the form

$$\Delta(X_i) = X_i \otimes C + u_i^j \otimes X_j. \qquad (5)$$

The quantum flip operator σ is related to the adjoint action of the elements u_i^j:

$$\mathrm{ad}\, u_i^j(X_k) = \sigma_{ik}^{lj} X_l. \qquad (6)$$

We will illustrate the constructions in this paper by the case of $\mathbf{g} = \mathbf{su}(2)$, for which the quantum Lie algebra $\mathbf{su}(2)_q$ has a basis $\{X_0, X_+, X_-\}$ with brackets

$$\begin{array}{lll}
[X_+, X_+] = 0, & [X_+, X_0] = -q^{-1}X_+, & [X_+, X_-] = (q + q^{-1})X_0, \\
[X_0, X_+] = qX_+, & [X_0, X_0] = (q - q^{-1})X_0, & [X_0, X_-] = -q^{-1}X_-, \\
[X_-, X_+] = -(q + q^{-1})X_0, & [X_-, X_0] = qX_-, & [X_-, X_-] = 0,
\end{array}$$
$$(7)$$

central element C given by

$$C^2 = 1 + (q - q^{-1})^2 \left(X_0^2 + \frac{qX_-X_+ + q^{-1}X_+X_-}{q + q^{-1}} \right), \qquad (8)$$

and relations

$$\begin{aligned}
qX_0X_+ - q^{-1}X_+X_0 &= CX_+, \\
q^{-1}X_0X_- - qX_-X_0 &= -CX_-, \\
X_+X_- - X_-X_+ + (q^2 - q^{-2})X_0^2 &= (q + q^{-1})CX_0.
\end{aligned} \qquad (9)$$

We would like to make the quantum group $\mathcal{H} = U_q(\mathbf{g})$ into a gauge group by taking gauge transformations to be elements $h(x)$ of $\mathcal{H}$ depending on the space-time point x, i.e. polynomials (or power series) in the Lie algebra elements X_i with x-dependent coefficients. A gauge potential $A_\mu(x)$ should be a space-time vector field with values in the quantum Lie algebra $\mathbf{g}_q$:

$$A_\mu(x) = A_\mu^i(x)X_i \qquad (10)$$

where the $A_\mu^i(x)$ are ordinary fields. It should transform under the gauge transformation $h(x)$ by

$$A_\mu(x) \mapsto A_\mu'(x) = h(x)_{(1)} A_\mu(x) S\left(h(x)_{(2)} \right) - \alpha^{-1} S(C)^{-1} \partial_\mu \left(h(x)_{(1)} \right) S\left(h(x)_{(2)} \right) \qquad (11)$$

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where α is a coupling constant. However, the second term here is not well-defined, since we do not know how to distribute the x-dependence of the co-product $\Delta(h(x))$ between the two factors before differentiating the first factor. Moreover, the set of gauge transformations $h(x)$ with values in $\mathcal{H}$ would be very much bigger than the classical gauge group of functions $g(x)$ with values in the group G whose Lie algebra is $\mathbf{g}$. Such functions can be written as $g(x) = \exp X(x)$ where $X(x)$ is a function with values in the Lie algebra $\mathbf{g}$. We therefore restrict ourselves to gauge transformations of the form

$$h(x) = f(X(x)) \tag{12}$$

where f is a polynomial or power series function and $X(x)$ is a function on space-time with values in the quantum Lie algebra $\mathbf{g}_q$:

$$X(x) = \xi^i(x)X_i. \tag{13}$$

We define the coproducts of such infinitesimal gauge transformations by

$$\Delta(X(x)) = \xi^i(x)X_i \otimes C + u_i^j \otimes \xi^i(x)X_j \tag{14}$$

(see (5)), and extend to powers of $X(x)$ multiplicatively. Then if $h(x)$ is as in (12) we can form

$$m \circ (\partial_\mu \otimes S)\Delta h(x) = \sum \partial_\mu \left(h(x)_{(1)}\right) S \left(h(x)_{(2)}\right)$$

where $m : \mathcal{H} \otimes \mathcal{H} \to \mathcal{H}$ denotes multiplication in $\mathcal{H}$. (What we are doing is to regard X as an element of $\mathcal{H}(\cdot) = \mathcal{F} \otimes \mathcal{H}$ where $\mathcal{F}$ is an algebra of c-number functions on space-time; then we have defined $\Delta X(\cdot)$ and $\Delta h(\cdot)$ as elements of $\mathcal{H}(\cdot) \otimes \mathcal{H}(\cdot)$ and we can apply $\partial_\mu \otimes S$ to $\Delta h(\cdot)$ before multiplying the two factors.) Define

$$\delta_\mu h(x) = S(C)^{-1}m \circ (\partial_\mu \otimes S)\Delta h(x).$$

We will show that $\delta_\mu h(x)$ belongs to the quantum Lie algebra $\mathbf{g}_q$.

We consider powers of infinitesimal gauge transformations, $h_n(x) = X(x)^n$, and proceed by induction on n. For $n = 1$ we have

$$h_1(x) = \xi^i(x)X_i,$$

$$\begin{aligned}
\delta_\mu h_1(x) &= S(C)^{-1}m \circ (\partial_\mu \otimes S) \left(\xi^i(x) \otimes C + u_i^j \otimes \xi^i(x)X_j\right) \\
&= S(C)^{-1}\partial_\mu\xi^i X_i S(C).
\end{aligned}$$

But $S(C)$ is central if C is, for

$$\begin{aligned}
S(C)x &= x_{(1)}S(C)S(x_{(2)})x_{(3)} = x_{(1)}S(x_{(2)}C)x_{(3)} = x_{(1)}S(Cx_{(2)})x_{(3)} \\
&= x_{(1)}S(x_{(2)})S(C)x_{(3)} = \varepsilon(x_{(1)})S(C)x_{(2)} = S(C)x.
\end{aligned}$$

Hence
$$\delta_\mu h_1(x) = \partial_\mu \xi^i(x) X_i \in \mathbf{g}_q. \tag{15}$$

Now suppose $\delta_\mu h_m(x)$ and $\delta_\mu h_n(x)$ both belong to $\mathbf{g}_q$, and consider the product $h_{m+n}(x) = h_m(x)h_n(x)$. We have

$$
\begin{aligned}
\delta_\mu h_{m+n}(x) &= S(C)^{-1} m \circ (\partial_\mu \otimes S) \left(h_m(x)_{(1)} h_n(x)_{(1)} \otimes h_m(x)_{(2)} h_n(x)_{(2)} \right) \\
&= S(C)^{-1} \left(\partial_\mu h_m(x)_{(1)} h_n(x)_{(1)} + h_m(x)_{(1)} \partial_\mu h_n(x)_{(1)} \right) \\
&\quad S\left(h_n(x)_{(2)} \right) S\left(h_m(x)_{(2)} \right) \\
&= \varepsilon(h_n(x)) \delta_\mu h_m(x) + \mathrm{ad} h_m(x).\delta_\mu h_n(x)
\end{aligned}
$$

which belongs to $\mathbf{g}_q$ since $\mathbf{g}_q$ is ad-invariant.

It follows that the transformation law (11) is well-defined for gauge transformations of the type (12), i.e. that $A'_\mu(x)$ is a gauge potential (belongs to $\mathbf{g}_q$) if $A_\mu(x)$ is.

3   Covariant Derivatives and Field Strengths

Let Ψ be a multiplet of fields transforming according to a representation ρ of the Hopf algebra $\mathcal{H}$. Associated with this representation are the scalars

$$c_\rho = \rho(C), \qquad\qquad \widetilde{c}_\rho = \rho(S(C)).$$

We define the covariant derivative

$$D_\mu \Psi = \partial_\mu \Psi + \alpha \widetilde{c}_\rho \rho(A_\mu) \Psi. \tag{16}$$

Assume that a product of fields transforms under a gauge transformation $h(x)$ according to the coproduct in $\mathcal{H}$: if fields $\phi(x), \psi(x)$ transform by

$$\phi(x) \mapsto T_\phi[h(x), \phi(x)], \qquad \psi(x) \mapsto T_\psi[h(x), \psi(x)]$$

then the product $\phi(x)\psi(x)$ transforms by

$$\phi(x)\psi(x) \mapsto T_\phi[h(x)_{(1)}, \phi(x)] T_\psi[h(x)_{(2)}, \psi(x)]. \tag{17}$$

Then the covariant derivative transforms by

$$
\begin{aligned}
D_\mu \Psi(x) &\mapsto \partial_\mu[\rho(h(x))\Psi(x)] + \alpha\rho\left(T_A[h(x)_{(1)}, A_\mu(x)] \right) T_\Psi[h(x)_{(2)}, \Psi(x)] \\
&= \partial_\mu \rho(h(x))\Psi(x) + \rho(h(x))\partial_\mu \Psi(x) \\
&\quad + \alpha \widetilde{c}_\rho \rho\left(h_{(1)} A_\mu S(h_{(2)}) - \alpha^{-1} S(C)^{-1} \partial_\mu h_{(1)} S(h_{(2)}) \right) \rho\left(h_{(3)} \right) \Psi \\
&= \rho(h(x)) D_\mu \Psi(x). \tag{18}
\end{aligned}
$$

Thus $D_\mu \Psi$ is a covariant derivative in the sense that it transforms in the same way as Ψ.

To define the field strengths, it is necessary to introduce a second bracket $\{,\}_q$ on the quantum Lie algebra $\mathbf{g}_q$ by

$$\{X_i, X_j\}_q = \tau_{ij}^{kl}[X_k, X_l]_q \tag{19}$$

where τ is an $N^2 \times N^2$ matrix satisfying

$$(\sigma - 1)(1 - c_0\tau + \tau\sigma) = 0, \tag{20}$$

c_0 being the value of the central element C in the N-dimensional adjoint representation ($c_0 = q^2 - 1 + q^{-2}$ for $\mathbf{g} = \mathbf{su}(n)$ [12]). This τ is arbitrary to the extent that one can add to it any solution τ' of

$$(\sigma - 1)\tau'(\sigma - c_0) = 0$$

but in general (i.e. if c_0 is not an eigenvalue of σ) this will imply $\sigma\tau' = \tau'$ and therefore by (4) the bracket $\{,\}_q$ of (19) will be unchanged. In the case of $\mathbf{su}(2)_q$, when σ has just the two eigenvalues 1 and $-c_0$, eq. (20) is satisfied by $\tau = (2c_0)^{-1}$ and so in this case the new bracket is just a multiple of the quantum Lie bracket:

$$\{X, Y\}_q = \frac{[X, Y]_q}{2c_0}. \tag{21}$$

The field strengths are now defined by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \alpha\Big(\{A_\mu, A_\nu\}_q - \{A_\nu, A_\mu\}_q\Big). \tag{22}$$

To investigate their transformation properties, we need only consider an infinitesimal gauge transformation $h(x) = \xi^i(x)X_i$ since the general gauge transformation is built up from these. For such $h(x)$ the transformation law (11) for the potentials becomes

$$A_\mu(x) \mapsto [h(x), A_\mu(x)]_q - \alpha^{-1}\partial_\mu h(x)$$

so that

$$\{A_\mu(x), A_\nu(x)\}_q \mapsto \{\mathrm{ad}h_{(1)}(A_\mu) - \alpha^{-1}\delta_\mu h_{(1)}, \ \mathrm{ad}h_{(2)}(A_\nu) - \alpha^{-1}\delta_\nu h_{(2)}\}_q$$

$$= \mathrm{ad}h(x)\{A_\mu, A_\nu\}_q - \alpha^{-1}\{\partial_\mu h(x), c_0 A_\nu\}_q - \alpha^{-1}\{\mathrm{ad}u_i^j(A_\mu), \partial_\nu \xi^i X_j\}_q$$

using the coproducts (5) again, and the fact that for the form of $h(x)$ we are considering $h(x)_{(1)}$ and $h(x)_{(2)}$ are not both x-dependent in any term of the coproduct. Using eq. (6) for $\mathrm{ad}u_i^j$ and the definition (19–20) of the bracket $\{,\}_q$, we find that the field strengths transform as they ought to, like ordinary matter fields in the adjoint representation:

$$F_{\mu\nu} \mapsto F'_{\mu\nu} = \mathrm{ad}h(x)F_{\mu\nu}. \tag{23}$$

It follows as usual that a gauge-invariant Lagrangian can be constructed by forming a function of $F_{\mu\nu}$, Ψ and $D_\mu\Psi$ which is invariant under constant transformations by elements of $\mathcal{H}$. Such a function of the field strengths alone, for example, is

$$\mathcal{L}_F = t_{ij} F^i_{\mu\nu} F^{j\mu\nu} \tag{24}$$

where

$$t_{ij} = \operatorname{tr}(T \operatorname{ad} X_i \operatorname{ad} X_j),$$

$T = \sum S(\mathcal{R}_2)\mathcal{R}_1$ being the quantum trace element of $\mathcal{H}$ and $\mathcal{R} = \sum \mathcal{R}_1 \otimes \mathcal{R}_2$ its universal R-matrix. For $\mathcal{H} = U_q(\mathbf{su}(2))$ this Lagrangian is

$$\mathcal{L}_F = (q + q^{-1})(qF^-.F^+ + q^{-1}F^+.F^-) + F_0^2. \tag{25}$$

4 Commutation Relations

Doubts might be felt about our assumption of (17) for the transformation properties of products of fields. In a quantum field theory, where symmetry transformations are implemented by unitary operators, there is no problem: if the gauge transformation $h(x)$ corresponds to an operator $U(h)$ and fields transform by

$$\psi(x) \mapsto U(h_{(1)})\psi(x)U(Sh_{(2)}), \tag{26}$$

then the transformation law for products is a consequence of this. In a classical theory this is not so clear; moreover, the transformation law for products may not be consistent with the fact that the components of a classical field should commute. In a quantum field theory this can be allowed for by changing the commutation relations of the field components.

Let $\Psi^i_\alpha(x)$ be a multiplet of fields transforming according to a representation ρ of the Hopf algebra $\mathcal{H}$:

$$U(h_{(1)})\Psi^i_\alpha(x)U(Sh_{(2)}) = \Psi^j_\alpha(x)\rho^i_j(h(x)) \tag{27}$$

α being a space-time (tensorial or spinorial) index. In the undeformed theory these fields will satisfy commutation relations of the form

$$[\Psi^i_\alpha(x), \Psi^j_\beta(y)] = \Delta^{ij}_{\alpha\beta}(x-y) \tag{28}$$

if the fields Ψ^i_α are bosonic (the fermionic case can be treated in a similar way). We can separate this into two equations, one symmetric and one antisymmetric in space-time indices:

$$\left[\Psi^i_\alpha(x)\Psi^j_\beta(y) \pm \Psi^i_\beta(y)\Psi^j_\alpha(x)\right]\Pi^{\pm kl}_{ij} = \left(\Delta^{ij}_{\alpha\beta} \pm \Delta^{ij}_{\beta\alpha}\right)\Pi^{\pm kl}_{ij} \tag{29}$$

where $\Pi^{\pm kl}_{ij} = \delta^k_i\delta^l_j \pm \delta^l_i\delta^k_j$ are the projectors onto the symmetric and antisymmetric subspaces of the tensor product $V \otimes V$ of the space V carrying the representation

ρ. These equations can easily be made compatible with the quantum group transformations; it is only necessary to replace $\Pi_{ij}^{\pm kl}$ by the projectors onto the corresponding invariant subspaces under the action of the quantum group (the q-symmetric and q-antisymmetric subspaces).

For the gauge potentials A_μ^i, the inhomogeneous term in the transformation (11) spoils this compatibility. In order to restore it for one of the sets of commutation relations, it is necessary to make the coupling constant α an operator with the commutation relation

$$\alpha A_\mu^i = c_0 A_\mu^i \alpha. \tag{30}$$

Then the effect of the transformation (11) on the commutator

$$\left[A_\mu^i(x) A_\nu^j(y) \mp A_\nu^i(y) A_\mu^j(x) \right] \Pi_{ij}^{\pm kl}$$

is to produce a term quadratic in A to which the same considerations apply as to a matter multiplet, the linear term

$$-\Big(A_\mu^m(x)[\mathrm{ad}\, h(x)_{(1)}]_m^i \alpha^{-1}[\delta_\nu h(y)_{(2)}]^j + \alpha^{-1}[\delta_\mu h(x)_{(1)}]^i A_\nu^m(y)[\mathrm{ad}\, h(y)_{(2)}]_m^j$$
$$\mp (\mu \leftrightarrow \nu, x \leftrightarrow y) \Big) \Pi_{ij}^{\pm kl}$$

and an inhomogeneous term

$$\alpha^{-2} \left([\delta_\mu h(x)_{(1)}]^i [\delta_\nu h(y)_{(2)}]^j \mp (\mu \leftrightarrow \nu, x \leftrightarrow y) \right) \Pi_{ij}^{\pm kl}.$$

If we consider an infinitesimal gauge transformation $h(x) = \xi^i(x) X_i$, the inhomogeneous term vanishes since $h_{(1)}$ and $h_{(2)}$ are not non-constant simultaneously, and the linear term becomes

$$c_0 \alpha^{-1} \left[A_\mu^m(x) \partial_\nu \xi^n(y) - A_\nu^m(y) \partial_\mu \xi^n(x) \right] (\sigma_{lk}^{ij} \mp \delta_l^i \delta_k^j) \Pi_{ij}^{\pm kl}.$$

For the adjoint representation ($V = \mathbf{g}_q$) the q-symmetric and q-antisymmetric subspaces of $V \otimes V$ are respectively the kernel and the image of the quantum flip σ, so $(\sigma - 1)\Pi^+ = 0$. Thus a commutation relation

$$\left[A_\mu^i(x) A_\nu^j(y) - A_\nu^i(y) A_\mu^j(x) \right] \Pi_{ij}^{+kl} = \text{c-number} \tag{31}$$

is invariant under the gauge transformation (11). However, σ does not have -1 as an eigenvalue if $q \neq 1$, and so the commutation relation

$$\left[A_\mu^i(x) A_\nu^j(y) + A_\nu^i(y) A_\mu^j(x) \right] \Pi_{ij}^{-kl} = \text{c-number} \tag{32}$$

is not gauge-invariant.

9

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